

Questão 4: 20min

(a) Feito

(b) $f(x) = x, a \in \mathbb{R}$

$$x-a \neq 0$$

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{x \rightarrow a} \frac{x - a}{x - a} \stackrel{\downarrow}{=} \lim_{x \rightarrow a} 1 = 1$$

logo, $D_x(x) = 1$

$$\text{Dom} \left(\frac{x-a}{x-a} \right)$$

(c) $f(x) = x^2, a \in \mathbb{R}$

$$x-a \neq 0$$

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$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{x \rightarrow a} \frac{x^2 - a^2}{x - a} \stackrel{\uparrow}{=} \lim_{x \rightarrow a} \frac{(x-a)(x+a)}{x-a} \stackrel{\uparrow}{=} \lim_{x \rightarrow a} x+a = a+a = 2a$$

$$\text{Dom} \left(\frac{x^2-a^2}{x-a} \right)$$

logo, $D_x(x^2) = 2x$

(d) $f(x) = x^n, a \in \mathbb{R}$

$$x-a \neq 0$$

$$f'(a) = \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = \lim_{x \rightarrow a} \frac{(x-a)(x^{n-1} + x^{n-2}a + \dots + xa^{n-2} + a^{n-1})}{x-a} \stackrel{\uparrow}{=}$$

$$\text{Dom} \left(\frac{x^n-a^n}{x-a} \right)$$

$$= \lim_{x \rightarrow a} x^{n-1} + x^{n-2}a + \dots + xa^{n-2} + a^{n-1} = \underbrace{a^{n-1} + a^{n-2}a + \dots + aa^{n-2} + a^{n-1}}_{n \text{ parcelas}} =$$

$$= n \cdot a^{n-1}$$

logo, $D_x(x^n) = n \cdot x^{n-1}$