

(e) $f(x) = \frac{1}{x^n}$, $a \in \mathbb{R} - \{0\}$

$$\begin{aligned}
 f'(a) &= \lim_{x \rightarrow a} \frac{\frac{1}{x^n} - \frac{1}{a^n}}{x-a} = \lim_{x \rightarrow a} \frac{\frac{a^n - x^n}{x^n a^n}}{x-a} = \lim_{x \rightarrow a} \frac{-(x^n - a^n)}{x-a} \cdot \frac{1}{x^n a^n} = \\
 &= \lim_{x \rightarrow a} \frac{-(x-a)(x^{n-1} + x^{n-2}a + \dots + xa^{n-2} + a^{n-1})}{(x-a)x^n a^n} \stackrel{x-a \neq 0}{=} \lim_{x \rightarrow a} \frac{-(x^{n-1} + x^{n-2}a + \dots + xa^{n-2} + a^{n-1})}{x^n a^n} \\
 &= \lim_{x \rightarrow a} \frac{-(x^{n-1} + x^{n-2}a + \dots + xa^{n-2} + a^{n-1})}{x^n a^n} = \frac{-(a^{n-1} + a^{n-1} + \dots + a^{n-1})}{a^n a^n} = \\
 &= \frac{-n a^{n-1}}{a^{2n}} = -n \cdot a^{(n-1)-2n} = -n a^{-n-1}
 \end{aligned}$$

n -parcelas

logo; $\mathcal{D}_x(x^{-n}) = -n x^{-n-1}$

(f) $f(x) = \sqrt{x}$, $a \in (0, +\infty)$

$$\begin{aligned}
 f'(a) &= \lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a}}{x-a} = \lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a}}{x-a} \cdot \frac{(\sqrt{x} + \sqrt{a})}{(\sqrt{x} + \sqrt{a})} = \lim_{x \rightarrow a} \frac{x-a}{(x-a)(\sqrt{x} + \sqrt{a})} \stackrel{x-a \neq 0}{=} \lim_{x \rightarrow a} \frac{1}{\sqrt{x} + \sqrt{a}} \\
 &= \lim_{x \rightarrow a} \frac{1}{\sqrt{x} + \sqrt{a}} = \frac{1}{\sqrt{a} + \sqrt{a}} = \frac{1}{2\sqrt{a}} = \frac{1}{2} a^{-\frac{1}{2}}
 \end{aligned}$$

