

(g) $f(x) = \sqrt[n]{x}$, $a \in \text{Dom} f - \{0\}$

$u \neq 0$
↑

$$f'(a) = \lim_{x \rightarrow a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a} = \lim_{u \rightarrow b} \frac{u - b}{u^n - b^n} = \lim_{u \rightarrow b} \frac{u - b}{(u - b)(u^{n-1} + u^{n-2}b + \dots + ub^{n-2} + b^{n-1})} =$$

$$\begin{cases} u = \sqrt[n]{x} \Leftrightarrow x = u^n \\ b = \sqrt[n]{a} \Leftrightarrow a = b^n \\ x \rightarrow a \Rightarrow u \rightarrow b \end{cases}$$

$$= \lim_{u \rightarrow b} \frac{1}{u^{n-1} + u^{n-2}b + \dots + ub^{n-2} + b^{n-1}} = \frac{1}{n \cdot b^{n-1}} = \frac{1}{n} \cdot b^{1-n} = \frac{1}{n} \cdot (a^{\frac{1}{n}})^{1-n} =$$

$$= \frac{1}{n} (a^{\frac{1-n}{n}}) = \frac{1}{n} a^{\frac{1}{n} - 1}$$

Logo, $D_x(\sqrt[n]{x}) = \frac{1}{n} x^{\frac{1}{n} - 1}$

Questão 5:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\lim_{x \rightarrow 0^+} |x| = \lim_{x \rightarrow 0^+} x = 0$$

$$\lim_{x \rightarrow 0^-} |x| = \lim_{x \rightarrow 0^-} -x = 0$$

$\left. \begin{array}{l} \lim_{x \rightarrow 0^+} |x| = \lim_{x \rightarrow 0^-} |x| = 0 \\ \lim_{x \rightarrow 0} |x| = |0| \end{array} \right\} \Rightarrow |x| \text{ é contínua em } x=0$

$|0| = 0$

$$\lim_{x \rightarrow 0^+} \frac{|x| - |0|}{x - 0} = \lim_{x \rightarrow 0^+} \frac{|x|}{x} = \lim_{x \rightarrow 0^+} \frac{x}{x} \stackrel{x \neq 0}{=} \lim_{x \rightarrow 0^+} 1 = 1$$

$$\lim_{x \rightarrow 0^-} \frac{|x| - |0|}{x - 0} = \lim_{x \rightarrow 0^-} \frac{|x|}{x} = \lim_{x \rightarrow 0^-} \frac{-x}{x} = \lim_{x \rightarrow 0^-} -1 = -1$$

$\Rightarrow \nexists \lim_{x \rightarrow 0} \frac{|x| - |0|}{x - 0} \Rightarrow \nexists f'(0)$