

(1º) Assintotas horizontais de $\tanh x$

$$\lim_{x \rightarrow +\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = \lim_{x \rightarrow +\infty} \frac{e^x (1 - e^{-2x})}{e^x (1 + e^{-2x})} = \lim_{x \rightarrow +\infty} \frac{1 - \frac{1}{e^{2x}}}{1 + \frac{1}{e^{2x}}} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = \lim_{x \rightarrow -\infty} \frac{e^{-x} (e^{2x} - 1)}{e^{-x} (e^{2x} + 1)} = \lim_{x \rightarrow -\infty} \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{-1}{1} = -1$$

(2º) $\lim_{x \rightarrow -\infty} \tanh \left(\frac{4x^3 - 2}{|1 - 2x|} \right)$

$$\lim_{x \rightarrow -\infty} \frac{4x^3 - 2}{|1 - 2x|} = \lim_{x \rightarrow -\infty} \frac{x^3 (4 - \frac{2}{x^3})}{|x(\frac{1}{x} - 2)|} = \lim_{x \rightarrow -\infty} \frac{x^3 (4 - \frac{2}{x^3})}{|x| |\frac{1}{x} - 2|} =$$

$$= \lim_{x \rightarrow -\infty} \frac{x^3 (4 - \frac{2}{x^3})}{-x |\frac{1}{x} - 2|} = \lim_{x \rightarrow -\infty} \frac{-x^2 (4 - \frac{2}{x^3})}{|\frac{1}{x} - 2|} = -(-\infty)^2 \frac{4}{1 - 2} =$$

$x \rightarrow -\infty$
 $x < 0 \Rightarrow |x| = -x$

$$= -(+\infty) \cdot (+2) = -\infty$$

loop; $\lim_{x \rightarrow -\infty} \tanh \left(\frac{4x^3 - 2}{|1 - 2x|} \right) = -1$

\downarrow
 $\lim_{y \rightarrow -\infty} \tanh(y) = -1$ (1º)