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(c) $$\lim_{x\to+\infty}\frac{P(x)}{Q(x)}=\lim_{x\to+\infty}\frac{x^n\left(a_n+\frac{a_{n-1}}{x}+\dots+\frac{a_1}{x^{n-1}}+\frac{a_0}{x^n}\right)}{x^m\left(b_m+\frac{b_{m-1}}{x}+\dots+\frac{b_1}{x^{m-1}}+\frac{b_0}{x^m}\right)}=$$

$$=\begin{cases}\displaystyle\lim_{x\to+\infty}\frac{a_n+\frac{a_{n-1}}{x}+\dots+\frac{a_1}{x^{n-1}}+\frac{a_0}{x^n}}{b_n+\frac{b_{n-1}}{x}+\dots+\frac{b_1}{x^{n-1}}+\frac{b_0}{x^n}}=\frac{a_n}{b_n} & \text{Use } n=m\\[2ex] \displaystyle\lim_{x\to+\infty}\frac{1}{x^{m-n}}\frac{\left(a_n+\frac{a_{n-1}}{x}+\dots+\frac{a_1}{x^{n-1}}+\frac{a_0}{x^n}\right)}{\left(b_m+\frac{b_{m-1}}{x}+\dots+\frac{b_1}{x^{m-1}}+\frac{b_0}{x^m}\right)}=0\cdot\frac{a_n}{b_m}=0 & \text{Use } m>n\\[2ex] \displaystyle\lim_{x\to+\infty}x^{n-m}\frac{\left(a_n+\frac{a_{n-1}}{x}+\dots+\frac{a_1}{x^{n-1}}+\frac{a_0}{x^n}\right)}{\left(b_m+\frac{b_{m-1}}{x}+\dots+\frac{b_1}{x^{m-1}}+\frac{b_0}{x^m}\right)}=\underbrace{(+\infty)^{n-m}\frac{a_n}{b_m}}_{=(+\infty)\cdot\frac{a_n}{b_m}} & \text{Use } m<n\end{cases}$$

(d) $$\lim_{x\to-\infty}\frac{P(x)}{Q(x)}=\begin{cases}\displaystyle\lim_{x\to-\infty}\frac{a_n+\frac{a_{n-1}}{x}+\dots+\frac{a_1}{x^{n-1}}+\frac{a_0}{x^n}}{b_n+\frac{b_{n-1}}{x}+\dots+\frac{b_1}{x^{n-1}}+\frac{b_0}{x^n}}=\frac{a_n}{b_n} & \text{Use } n=m\\[2ex] \displaystyle\lim_{x\to-\infty}\frac{1}{x^{m-n}}\cdot\frac{a_n+\frac{a_{n-1}}{x}+\dots+\frac{a_1}{x^{n-1}}+\frac{a_0}{x^n}}{b_m+\frac{b_{m-1}}{x}+\dots+\frac{b_1}{x^{m-1}}+\frac{b_0}{x^m}}=0\cdot\frac{a_n}{b_m}=0 & \text{use } m>n\\[2ex] \displaystyle\lim_{x\to-\infty}x^{n-m}\frac{a_n+\frac{a_{n-1}}{x}+\dots+\frac{a_1}{x^{n-1}}+\frac{a_0}{x^n}}{b_m+\frac{b_{m-1}}{x}+\dots+\frac{b_1}{x^{m-1}}+\frac{b_0}{x^m}}=(-\infty)^{n-m}\frac{a_n}{b_m} & \text{use } m<n\end{cases}$$

Questão 12: Livre (verificar se atende os requisitos do item)